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ELECTRON DIFFUSION IN A TURBULENT PLASMA

By Harold R. Kaufman

Lewis Research Center
Cleveland, Ohio

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2

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SUMMARY

Experiments with a fluctuating or turbulent plasma have shown diffusion rates drastically different from the nonfluctuating or laminar values. This large difference between experiment and theory demonstrated the need for even a qualitative analysis of electron diffusion in a turbulent plasma. The conditions at which instability is encountered depend on the type of instability assumed. Two-stream instability was selected as the turbulence-producing mechanism in this analysis not only because of its probable importance, but because considerable information is available on this type of instability.

As the plasma changes from a laminar to a fully developed turbulent state, the electron mean free path should decrease from the laminar to some minimum turbulent value, which is determined by energy-dissipating processes in the plasma. The transitional region is defined by mean free paths between these two extremes. For the two-stream instability mechanism used herein, the onset of turbulence should occur when the electron drift velocity (ordered relative motion between electrons and ions) reaches the critical value. The expected minimum mean free path for this mechanism should be roughly the minimum plasma wavelength, which is set by a rapid increase in damping. In the transitional region, the electron drift velocity should tend toward the critical value. The theoretical results were also compared with experimental data. Although the results of this comparison were not conclusive, the general experimental trends were apparently in agreement with those hypothesized.

INTRODUCTION

The Lewis electric propulsion research program includes a variety of ion and plasma devices. The diffusion of electrons in a plasma is a problem common to most of these devices. Electron diffusion in a non-fluctuating, or laminar, plasma has been adequately treated in existing literature (refs. 1 and 2). The treatment of a fluctuating plasma, however, has been less satisfactory.

E-1619

Experiments with a fluctuating plasma have shown diffusion rates drastically different from the nonfluctuating or laminar values. One such experiment at Lewis (ref. 3) gave an experimental result that differed from the laminar-theory value by a factor of 10^5 . This large difference between experiment and theory demonstrated the need for a qualitative analysis of electron diffusion in a fluctuating plasma.

Fluctuations in a plasma can come from a variety of external sources, such as anode and cathode phenomena. As described in reference 4, they can also be due to phenomena occurring within the plasma. It is to these latter fluctuations, which result primarily from mechanisms in the bulk of the plasma, that the present report is directed. A plasma exhibiting such fluctuations is called turbulent herein.

The result of an instability approach to the turbulent plasma problem is indicated in reference 4, although the derivation is not included. A quantum-mechanical approach, which shows that plasma waves increase electron diffusion, is presented in reference 5. Both these methods omit means of estimating the magnitude of turbulence that should be expected - or even whether the plasma should be laminar or turbulent. Another approach to the turbulent plasma problem, which indicates the onset of turbulence for conduction parallel to a magnetic field, is discussed in reference 6.

The conditions at which instability is encountered depend on the type of instability assumed. Two-stream instability was selected as the turbulence-producing mechanism in this analysis. The ordered relative motion of electrons and ions, when it reaches a critical value, can serve to amplify plasma-wave disturbances. This amplification of disturbances is called two-stream instability. This type of instability was selected not only because it probably is important, but also because a large amount of pertinent information is available in the literature. It is hoped, however, that the analysis may also serve to indicate qualitative trends for other types of instabilities.

Some simplifying assumptions were used in this analysis. The extent of the plasma was assumed large compared with the Debye shielding distance, which implies substantially equal electron and ion charge density. Singly ionized atoms and electrons with a uniform random or thermal velocity were also assumed.

The method of analysis used was to treat the mean or macroscopic diffusion process and to ignore the microscopic turbulent fluctuations. The results of the analysis are compared with available experimental data. As an aid to the reader, brief summaries of pertinent laminar-diffusion and plasma-wave equations are also included. A list of symbols is presented in the appendix. Rationalized mks units are used throughout.

LAMINAR PLASMA

Electron diffusion in a laminar or nonturbulent plasma is well understood. The material presented in this section was abstracted from references 1 and 2 and is included only for the convenience of the reader. In keeping with the turbulent-diffusion equations to follow, a uniform electron velocity was assumed for the laminar equations. The velocity used was the root-mean-square (rms) value. References 1 and 2 contain the more precise electron-diffusion equations, based on a Maxwellian distribution.

Without Magnetic Field

The electron current density (current in amperes) resulting from both density and potential gradients can be written

$$j = -qD_0 \frac{dn}{dx} + qn \frac{D_0}{V} \frac{dV}{dx} \quad (1)$$

Using the rms electron velocity yields

$$\bar{v} = \left(\frac{3q\bar{V}}{m} \right)^{1/2} = 7.27 \times 10^5 \bar{V}^{1/2} \quad (2)$$

The diffusion coefficient is defined by

$$D_0 = \frac{\bar{v}l}{3} = 2.42 \times 10^5 l \bar{V}^{1/2} \quad (3)$$

The conductivity is then

$$\sigma_0 = qn \frac{D_0}{V} = 3.88 \times 10^{-14} \frac{nl}{\bar{V}^{1/2}} \quad (4)$$

With Magnetic Field

The diffusion parallel to a magnetic field is the same as that in the absence of a magnetic field:

$$D_{||} = D_0 \quad (5)$$

The diffusion normal to a magnetic field, however, is reduced:

$$D_{\perp} = \frac{D_0}{1 + (l/r)^2} \quad (6)$$

The electron cyclotron radius based on rms velocity is

$$r = \frac{mv}{qB} = 4.13 \times 10^{-6} \frac{\bar{v}^{1/2}}{B} \quad (7)$$

and the effect of the magnetic field on ion motion is assumed to be negligible.

The motion of electrons normal to a magnetic field is not only in the direction of the applied gradient (diffusion), but also in a direction normal to both the magnetic field and the applied gradient. The combined (motion of the guiding centers) is thus at an angle θ with the applied gradient, with θ defined as

$$\theta = \arctan (l/r) \quad (8)$$

The current density at the angle θ can be shown, from geometrical considerations, to be

$$j_{\theta} = j_{\perp} \left[1 + (l/r)^2 \right]^{1/2} \quad (9)$$

where j_{\perp} is the diffusion current density (in the direction of the applied gradient).

The strong field approximation is obtained for $l/r \gg 1$. The diffusion coefficient normal to a magnetic field then becomes

$$D_{\perp} = D_0 (r/l)^2 = 4.13 \times 10^{-6} \frac{\bar{v}^{3/2}}{lB^2} \quad (10)$$

and the conductivity,

$$\sigma_{\perp} = qn \frac{D_{\perp}}{\bar{v}} = 6.62 \times 10^{-25} \frac{n \bar{v}^{1/2}}{lB^2} \quad (11)$$

The drift current density is

$$j_{\theta} = j_{\perp} (l/r) \quad (12)$$

Electron Mean Free Path

The mean-free-path length, or collision distance, for electrons in a laminar plasma is obtained from the collision cross sections of electrons with ions and neutrals. The collisions with ions are of both the distant, or coulomb, type and the near, or large-deflection, type, while those with neutrals are only of the near, or large-deflection, type. Because these various collisions are mutually exclusive, a simple summation gives the combined effect on electron mean free path:

$$l = \frac{1}{n \sum Q} \quad (13)$$

where

$$\sum Q = (Q_d + Q_n)_{\text{ions}} + \frac{n_o}{n} (Q_n)_{\text{neutrals}}$$

The cross section for distant collisions between electrons and ions, based on rms velocity, is

$$Q_d = \frac{q^2 \ln \Lambda}{18\pi\epsilon_o^2 \bar{V}^2} = 5.79 \times 10^{-18} \frac{\ln \Lambda}{\bar{V}^2} \quad (14)$$

The parameter Λ is defined as the quotient of Debye shielding length l_D and the impact parameter for a 90° deflection b , where

$$l_D = \left(\frac{\epsilon_o \bar{V}}{nq} \right)^{1/2} = 7.43 \times 10^3 \frac{\bar{V}^{1/2}}{n^{1/2}} \quad (15)$$

and

$$b = \frac{q}{8\pi\epsilon_o \bar{V}} = \frac{7.20 \times 10^{-10}}{\bar{V}} \quad (16)$$

Using $\sqrt{3}\bar{V}/2$ in place of \bar{V} gives

$$\Lambda = \frac{l_D}{b} = 1.55 \times 10^{13} \frac{\bar{V}^{3/2}}{n^{1/2}} \quad (17)$$

Values of $\ln \Lambda$ are presented in the following table for a range of electron temperature and density:

Electron density, n , $\frac{\text{number}}{\text{cu m}}$	$\ln \Lambda$			
	Electron temperature, \bar{V} , volts			
	1	10	100	1000
10^{12}	16.6	20.0	23.5	26.9
10^{15}	13.1	16.6	20.0	23.5
10^{18}	9.6	13.1	16.6	20.0
10^{21}	6.2	9.6	13.1	16.6

The large-deflection cross section for neutrals can be obtained from information such as presented in reference 7. Only the total cross sections for large-deflection collisions, which often correspond to substantially less than 90° deflections, are usually presented. Thus, a reduction in cross-section area must be made to correct the data to 90° deflections. At 1 electron volt, this correction is usually 10 percent or less, while at 10 electron volts the correction might be as much as 30 percent. At 100 electron volts the cross-section reduction would typically be 30 to 70 percent.

The large-deflection cross sections for ions are generally unavailable. For singly ionized heavy elements, the cross section should be of the same order as the neutral. For ions of light elements, though, the cross section could be much less than that of a neutral. Fortunately the large-deflection cross section of an ion is usually of minor importance in determining electron path length, so that an error in its calculation is not critical.

PLASMA WAVES

To be consistent with the use of two-stream instability as the turbulence producing mechanism, the fluctuations in a turbulent plasma are assumed to be longitudinal electrostatic plasma waves. The instability is simply the means by which noise is amplified into plasma waves. As described in reference 8, there are two types of such plasma waves that are produced by electron drift (ordered relative motion between electrons

and ions). One type is the electron-plasma wave, in which the changes occur so rapidly that the ions are effectively stationary. The frequency of this oscillation is

$$f = \frac{1}{2\pi} \left(\frac{nq^2}{\epsilon_0 m} \right)^{1/2} = 8.98 n^{1/2} \quad (18)$$

The other type is the ion-plasma wave, in which the changes are so slow that the electrons continually adjust to the Boltzmann distribution. For very short wavelengths, the ion-plasma frequency is

$$F_0 = f \left(\frac{m}{M} \right)^{1/2} \quad (19)$$

For long wavelengths, the frequency is

$$F^2 = \frac{F_0^2}{1 + (\lambda/2\pi l_D)^2} \quad (20)$$

The drift velocity must exceed a critical value before plasma waves are amplified. This critical value for electron-plasma waves is given in the analysis of reference 9 as about three-fourths of the rms electron velocity. The corresponding critical value for the amplification of ion-plasma waves is given in reference 10 and depends on ion random energy or temperature. For equal electron and ion temperatures, typical of high-pressure discharges, the critical value is about the same as for electron-plasma waves. For ion temperatures much less than electron temperature, typical of low-pressure discharges, the critical drift velocity is lower by the square root of the electron-ion mass ratio.

The critical drift velocity can also be expressed as a current density:

$$j_c = \beta nq\bar{v} = 1.16 \times 10^{-13} \beta n\bar{v}^{1/2} \quad (21)$$

It should be evident from the preceding paragraph that the critical drift ratio β should be about 3/4 for equal electron and ion temperatures and about $(3/4)(m/M)^{1/2}$ for very low ion temperatures. The variation of β between these two limits is shown in reference 11. An approximate equation for this variation was found to be

$$\beta \approx 3e^{-\sqrt{2/\alpha}} \quad (22)$$

with α the ion-electron temperature ratio.

A typical time for electron-plasma waves to develop into random turbulence when the critical drift velocity is exceeded was found to be 30 electron-plasma periods in reference 9. The corresponding time for ion-plasma waves to develop has not been determined, but would be expected to be some similar number of ion-plasma periods.

Although a uniform random electron velocity (rms value) is assumed for the analysis to follow, a velocity distribution had to be assumed to calculate the preceding critical drift velocities. Of particular importance are the densities of electrons with velocities slightly greater and slightly less than plasma-wave phase velocity as these groups add to and detract from wave energy. Maxwellian distributions were assumed for the drift-velocity calculations of references 10 and 11, and it is quite certain that the results would have been markedly different for substantially nonMaxwellian distributions.

TURBULENT PLASMA

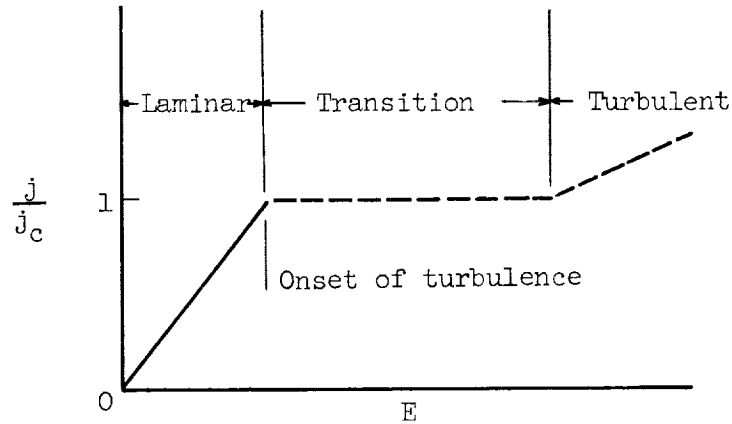
When the drift velocity (ordered relative motion of electrons and ions) exceeds the critical value, noise of the proper frequency (or range of frequencies) will be amplified into plasma waves of finite amplitude. These waves will deflect electrons and thus reduce the electron mean free path, or collision distance. Since the time for plasma waves to develop (or substantially change in amplitude) is far longer than the time for an electron to traverse a single wave, electron randomization should occur more rapidly than turbulence level changes. The turbulence level should therefore tend toward a value just sufficient to keep the drift velocity near the critical value. This proposed relation between drift velocity and turbulence, which results from a plasma instability, leads to what may be called a transition region between laminar and turbulent diffusion. The foregoing discussion assumes that the threshold of wave amplification is independent of wave amplitude. Such an assumption should be reasonable for a rough analysis.

The fully developed turbulent state is reached when large amplitude waves are produced over a broad range of frequencies. A large amount of the turbulent energy should then be found near the high-frequency limit, which should be set by some damping mechanism.

Without Magnetic Field

The idealized macroscopic effect of turbulence on electron diffusion in the absence of a magnetic field is shown in sketch (a):

E-1619



(a)

The plasma is assumed to be of constant density (ignoring turbulent fluctuations) with constant electron temperature. Also, the electric field in the plasma is assumed to change slowly, thus avoiding transient effects.

At small electric fields, the current varies linearly with electric field, as predicted by the laminar equations. Instability should be first encountered when the current density reaches the critical value ($j/j_c = 1$). The electric field at this point is

$$E = \frac{j_c}{\sigma_0} = \frac{3\beta\bar{V}}{l} \quad (23)$$

with l at the laminar value. In accordance with the discussion at the beginning of this section, the current density should vary only slightly in the transition region, as indicated in idealized form by the horizontal portion of the curve in sketch (a). The variation in electric field in this region serves primarily to change the turbulence level.

The electron path length should not be expected to become vanishingly small as the electric field is increased indefinitely. It would seem reasonable that the path length should not become smaller than the minimum turbulence scale which, for this particular model, is the minimum plasma wavelength. This minimum wavelength for either electron-plasma or ion-plasma waves is set by a rapid rise in damping at about

$$\lambda_{\min} = 2\pi l_D = 4.67 \times 10^4 \frac{\bar{V}^{1/2}}{n^{1/2}} \quad (24)$$

The diffusion expressions based on this electron path length are

$$D_0 = \frac{\bar{v}\lambda_{\min}}{3} = 1.13 \times 10^{10} \frac{\bar{v}}{n^{1/2}} \quad (25)$$

and

$$\sigma_0 = qn \frac{D_0}{\bar{v}} = 1.81 \times 10^{-9} n^{1/2} \quad (26)$$

The electric field at this point is then

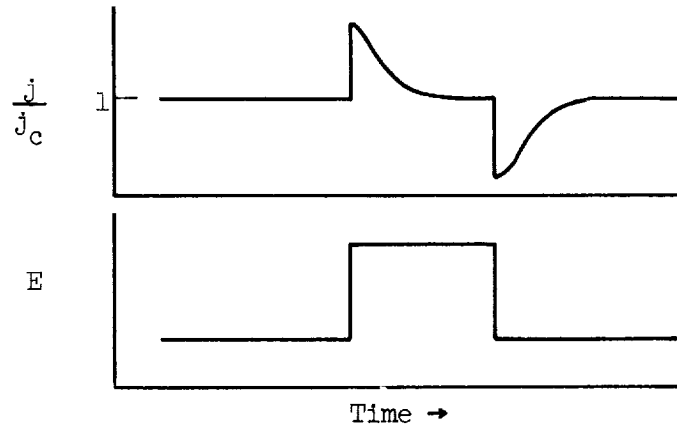
$$E = \frac{j_c}{\sigma_0} = 6.43 \times 10^{-5} \beta n^{1/2} \bar{v}^{1/2} \quad (27)$$

The current density might be expected to vary linearly with electric field beyond the minimum plasma wavelength point if the electron path length remains constant at λ_{\min} . Such a linear relation is indicated by the slope of the turbulent portion of the curve in sketch (a). A more rigorous interpretation might be that the limit of this particular analytical approach is reached when the electron path length is reduced to λ_{\min} .

In some experimental cases, the excitation and ionization processes may tend to keep electron temperature constant, as assumed. In many other cases, the electron temperature varies widely. The current in the transitional region should still be close to the critical value, as shown in sketch (a). The variation in critical current density (due to electron-temperature changes) may lead to a variation in absolute current - even though the ratio is nearly constant. Variations of ion and electron densities with electric field may lead to similar departures from the ideal relation between current and electric field. Here, too, the current should be nondimensionalized (by dividing with the critical value) before a comparison with sketch (a) is made.

The response to an electric-field transient can also be predicted, although in only a qualitative manner. Again a uniform plasma density and constant electron temperature were assumed, and the transient response of a plasma in the transitional region to a step change in electric field is shown in sketch (b):

E-1619



(b)

The turbulence level remains instantaneously constant as the electric field step is first imposed. Thus, the electron path length remains approximately constant, and the current increases proportionately with the electric field. This increased current increases the amplification of plasma waves, so that within a few plasma periods the turbulence level is increased and the electron path length decreased ($1/f$ or $1/F_0$ equals one period). The decrease in path length should tend to restore the current density to near the critical value.

Conversely, a sudden drop in electric field initially causes a decrease in current density below the critical value. Damping of the plasma waves at this lower current density will permit, within a few plasma periods, the current density to return to approximately its initial value. A plasma period, of course, refers to either electron-plasma or ion-plasma waves. Which one to expect will depend primarily on the ion-electron temperature ratio.

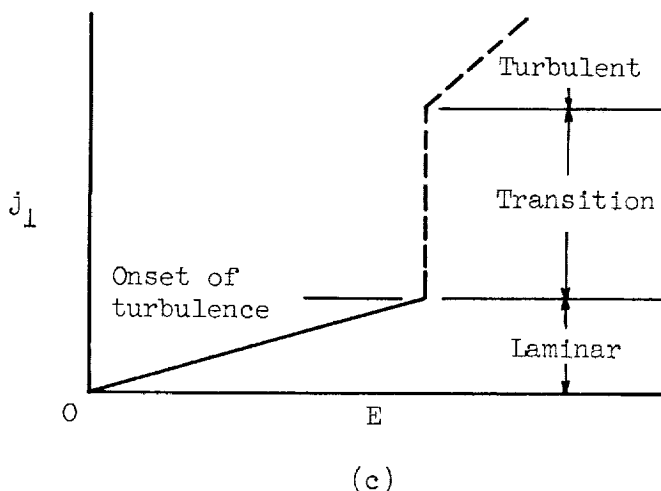
The preceding discussion applies when the temperature is constant. The results are more complex when the electron temperature is a strong function of current density, in that temperature changes can also cause absolute current changes.

With Magnetic Field

The strong magnetic field approximation will be considered first, again with uniform plasma density and constant electron temperature assumed. Inasmuch as the studies of references 9 to 11 assumed no magnetic field, it must also be assumed that the critical drift velocity is not a function of magnetic field. It should be mentioned again that the

two-stream mechanism is not the only likely type of instability in a turbulent plasma. The type discussed in reference 6 is particularly promising for certain geometries with current parallel to a magnetic field. For this analysis, however, the case of current parallel to a magnetic field is essentially the same as no magnetic field.

The electron path is constant at the laminar value for small electric fields, and the diffusion current therefore varies linearly with electric field, as shown in sketch (c):



In the transition region, the idealized behavior with a strong magnetic field is the inverse of that with no magnetic field. Small changes in electric field correspond to large changes in current, as indicated ideally by the vertical line in sketch (c). This result can be deduced by equating drift current density j_θ and critical current density j_c . From equation (12), the electron path length then becomes rj_c/j_\perp . Using this value for l gives the electric field in the transition region as

$$E = \frac{j_\perp}{\sigma_\perp} = \frac{3\beta\bar{V}}{r} \quad (28)$$

It is interesting that this result indicates an electric field proportional to B instead of B^2 as indicated by the laminar equations. Unfortunately the equivalent expression for diffusion coefficient contains the current density as a factor; therefore, a simple comparison with the turbulent diffusion expression in reference 4 (which also contains B instead of B^2) is impossible. The current density at the onset of

instability is obtained by setting the electron path length equal to the laminar value. Then, solving equation (10) for diffusion current density results in

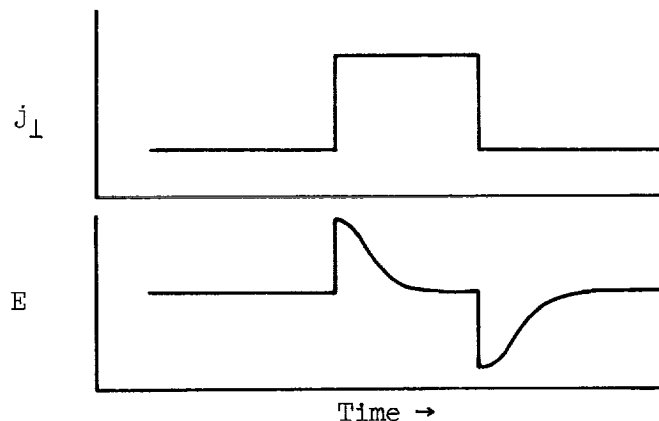
$$j_{\perp} = j_c (r/\lambda) \quad (29)$$

The current density at the start of the turbulent region is obtained in a similar manner by setting the electron path length equal to λ_{\min} :

$$j_{\perp} = j_c (r/\lambda_{\min}) \quad (30)$$

Beyond the start of the turbulent region, the diffusion is subject to the same uncertainties as in the absence of a magnetic field.

The transient response of a plasma in the transitional region with a strong magnetic field also differs substantially from that in the absence of a magnetic field. The transient response of the electric field with a current transient assumed to be imposed is shown in sketch (d):



(d)

Uniform plasma density and constant electron temperature are also assumed. The conductivity is initially unchanged as the step change in diffusion current is imposed, so that the electric field rises proportionately with current. Associated with the diffusion current increase, however, is a drift current increase (eq. (12)). This increase in drift current causes, within a few plasma periods, an increased turbulence and decreased electron path length. With both the electric field and drift current proportional to electron path length (eqs. (11) and (12)), the electric field tends to be restored to its initial value as the drift current returns to the critical value. A step decrease in diffusion

current causes the reverse of the process just described, with the electric field initially decreasing and the plasma turbulence damping to a lower value.

The complete expression for electron diffusion normal to a magnetic field is obtained by setting the drift current density in equation (9) equal to the critical value:

$$j_c = j_{\perp} \left[1 + (\ell/r)^2 \right]^{1/2} \quad (31)$$

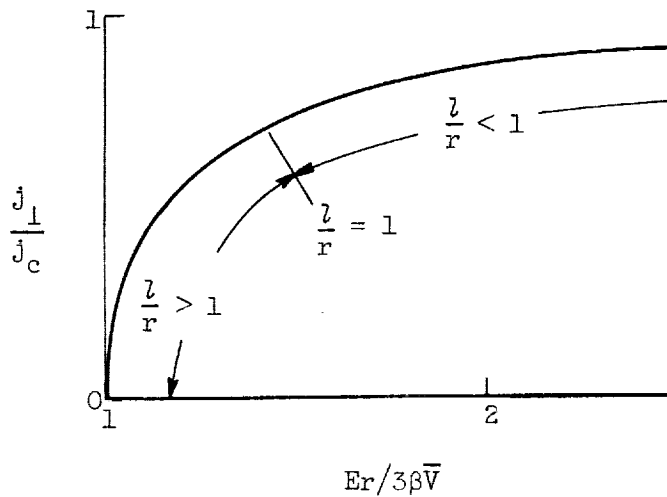
Solving for electron path length and substituting for that value in equation (6) result in

$$D_{\perp} = \frac{r\bar{v}}{3} \frac{\left[(j_c/j_{\perp})^2 - 1 \right]^{1/2}}{(j_c/j_{\perp})^2} \quad (32)$$

The electric field in the transitional region then becomes

$$E = \frac{3\beta\bar{v}}{r} \frac{j_c/j_{\perp}}{\left[(j_c/j_{\perp})^2 - 1 \right]^{1/2}} \quad (33)$$

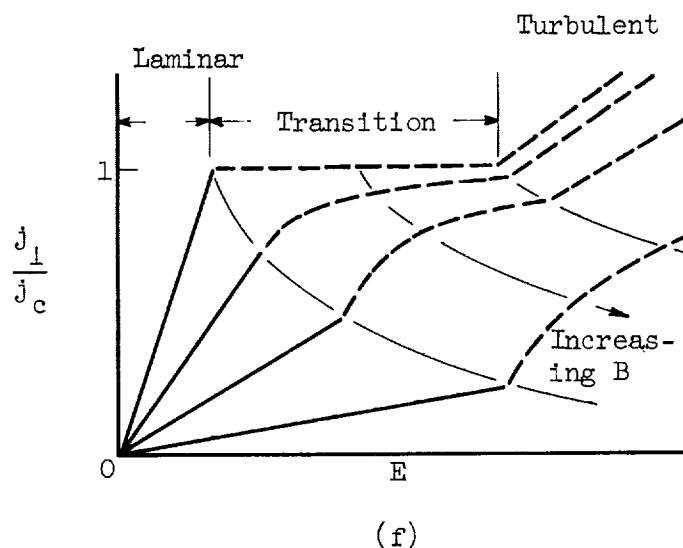
A nondimensional plot of current against electric field in the transitional region is shown in sketch (e):



(e)

At low values of j_{\perp}/j_c , the relation between current and electric field is similar to the strong magnetic field approximation with the electric field approximately constant for large current changes. At diffusion currents near the critical value, the behavior is similar to that in the absence of a magnetic field with large electric field changes corresponding to small current changes. The point where $l/r = 1$ corresponds to $j_{\perp}/j_c = 1/\sqrt{2}$.

The relation between current and electric field for a range of magnetic field strengths is shown in sketch (f):



Again, a uniform plasma density and a constant electron temperature are assumed. As shown in sketch (f), all the intermediate curve shapes between the no-magnetic-field case and the strong field approximation are obtained. At little or no magnetic fields, the diffusion current is approximately constant in the transitional region. At strong magnetic fields, the current density increases sharply at the onset of turbulence, so that a large change in current occurs with a small change in electric field.

COMPARISON WITH EXPERIMENT

The theory of turbulent electron diffusion is compared with data from four experiments (refs. 3, 12, 13, and 14) in this section. This comparison, although not conclusive, does give some indication of the validity of the turbulent diffusion theory.

Without Magnetic Field

An experiment is described in reference 3, in which the drift velocity exceeded the critical value. The plasma was the beam of an ion rocket operated in a large vacuum tank. The large drift velocity was obtained when the neutralizer was turned off and charge neutralization of the beam was accomplished by trapping stray electrons. The relative velocity between electrons and ions (drift velocity) under these conditions was very nearly equal to ion velocity. The ion random energy was believed to be negligible. With mercury as the ion-rocket propellant, the critical drift velocity ratio β should have been about 0.0012. The actual drift ratio (an electron temperature of 10 v was assumed) was about 0.02. The experiment therefore should have corresponded to the turbulent portion of sketch (a)

The experimental data indicated a gradient of about 10 volts per meter, while the maximum turbulence equation (eq. (26)) would have predicted a little over 100 volts per meter. For comparison, the laminar equation would have predicted only a few hundred microvolts per meter. Thus, the turbulent equation differed from the experimental result by about one order of magnitude, and the laminar equation differed by about five orders of magnitude. The drift velocity's exceeding the critical value by a factor of about 20 may be the cause of some of the discrepancy, inasmuch as the turbulent nature of the plasma would be quite uncertain. Also, although the ion beam was several meters long, the ion transit time was quite short; it corresponded to only 20 or 30 ion-plasma periods. The turbulence may not have been as fully developed as it would be with a longer transit time.

The ion rocket of reference 3 was also operated with the neutralizer on and gave zero net beam current. Two modes of operation were then observed. The first mode gave the expected result of a large reduction in both noise and potential gradient (by more than a factor of 10) - presumably because of a reduction in drift velocity. (The lowered noise and gradient in the first mode cannot be compared with laminar estimates because the noise signals approached background noise level, and the probe signals approached the level of work function uncertainty.) In the second mode, the noise and the gradient were about the same as those observed with the neutralizer off. The neutralizer bias was the only operating parameter that was different for the two modes, with the electrons being injected more energetically in the second mode.

It was deduced that the electrons in the beam fell into two energy groups for the second mode. One group would be the high-energy electrons from the neutralizer. These electrons might be only slightly randomized by the plasma waves and would be permitted to go directly to the target. The bulk of the neutralization would be with the second group, made up of low-energy trapped electrons. These low-energy electrons

would behave almost as if the neutralizer electrons did not exist. Thus, a nonMaxwellian distribution apparently had large effects on the turbulent diffusion process. The passage of neutralizer electrons through a turbulent plasma without randomization may be questioned. The variation of mean free path for high-energy electrons in a turbulent plasma, however, should be of the same form as that of coulomb collisions, with the mean free path increasing as the square of electron energy.

E-1619
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The second experiment to be compared with turbulent diffusion theory is described in reference 12. A discharge tube that was 3.6 centimeters in diameter and 60 centimeters long was filled with mercury vapor at a pressure of 7×10^{-3} millimeter. The drift ratio of electrons at the operating condition was 0.11, while the critical ratio should have been 0.0012. The mean time between ion collisions was about 10 to 20 ion-plasma periods, which was slightly less than in the first experiment. No turbulent fluctuations were observed and the potential gradient agreed closely with the value predicted by laminar equations. The lack of turbulence at a drift velocity many times the critical ratio illustrates two areas of doubt in turbulent diffusion theory.

First, particular care was used to reduce noise from the electrodes and power supply in this experiment. The turbulent theory assumes the existence of noise that can be amplified by the two-stream instability mechanism. The transition to a turbulent plasma should be delayed by a reduction in ambient noise. Present theory does not indicate the magnitude of this effect.

Second, although the random thermal motion of ions was quite small, the radial drift velocity was not. This mean radial drift velocity corresponded to almost 0.3 volt. The mean radial drift velocity would not be expected to have any effect inasmuch as the electrons would drift at about the same radial velocity. The spread in radial velocities, which would introduce a random radial velocity component of the same order as the mean radial drift velocity, should, however, be equivalent to increasing ion temperature. Using 0.3 volt for ion temperature yields an α of 0.2 instead of the previously assumed value of 0.02. Using an α of 0.2 in equation (22) gives a critical drift ratio of about 0.13, which is greater than the experimental value. Thus, the lack of turbulence may possibly be explained by the random ion motion resulting from the mean radial drift velocity.

With Magnetic Field

The first experiment with a magnetic field to be considered is the one described in reference 13. A discharge tube that was 2 centimeters in diameter and 4 meters long was operated with helium at a pressure of

1.46 millimeters of mercury. The axial magnetic field was varied from 0 to 0.53 webers per square meter (5300 gauss). Because of the high neutral density, the mean time between ion collisions was quite short; it amounted to only three to five ion-plasma periods. The electron and ion temperatures were about 3.5 and 0.09 volt, which indicates a critical drift ratio of about 0.01. The mean radial drift velocity of ions was estimated to be at least one-half of the ion thermal motion, which leads to a significant increase in transverse random motion.

Because of the large transverse random motion of electrons, as well as the short time between ion collisions, it is not surprising that the plasma was laminar at zero magnetic field even though the drift ratio of electrons was two to three times the critical value. When the magnetic field was increased to about 0.23 weber per square meter, however, the onset of instability was encountered.

Increasing the magnetic field reduces the electron diffusion to the tube wall, which decreases the radial potential gradient. This radial potential gradient is the principal cause of ion radial drift. In comparison with the zero magnetic field value of radial drift, the mean radial drift of ions at a field strength of 0.23 weber per square meter was estimated at only a few percent of the ion thermal motion. Further increases in magnetic field strength beyond 0.23 weber per square meter led to an increase in radial electron diffusion to over 10 times the laminar value. An increase in electron diffusion across a magnetic field would be expected from turbulent theory. The short time between ion collisions should have had a damping effect, which would tend to delay the turbulent transition. The data in reference 13, however, were not sufficient for either an estimate of the damping effect or agreement with turbulent diffusion equations.

The experimental data of reference 13 have also been explained by the analysis of reference 6, which is probably a better analytical approach when the radial drift velocities of ions are large compared with their thermal velocities. It is interesting, however, that both the approach of reference 6 and the two-stream approach indicate increasing instability with increasing radius and increasing axial drift velocity and thus indicate the same qualitative results.

The second experiment with a magnetic field to be considered is described in reference 14. Electrons are emitted at the center of one end of a cylinder that is 35 centimeters long. Anode diameters from 1 to 4 centimeters were used, with the anode forming the wall of the cylinder. Axial magnetic fields up to 0.4 weber per square meter were used. Electrons were prevented from escaping at the ends of the cylinder by reflectors at the emitter potential. Hydrogen at a pressure of 5×10^{-3} millimeter was used in the discharge. The potential difference

between anode and cathode (emitter and reflectors) was 280 volts. Surveys indicated the radial electric field was negligible in the center 1 centimeter of the discharge diameter. The potential difference between the anode and the center of the discharge ranged up to 190 volts, with the remainder of the 280 volts appearing between the discharge center and the cathode.

E-1619
With the large potential difference between the emitter and the center of the discharge, the electrons are injected at high velocity. These high-velocity electrons were adequately contained at high magnetic fields, so that the radial electric field could be compared with the strong field approximation (eq. (28)). The electron temperature outside of the center core was estimated at 5 to 10 volts, while the ion temperature was believed to be near that of the anode. With predominantly monatomic ions assumed present, the critical drift velocity ratio would be about 0.017. Operation was assumed to be in the transitional region (which was supported by noise measurements) and this ratio was used to compare experimental and theoretical radial electric fields. The best agreement with the data, however, was obtained by assuming a critical drift ratio up to twice as large as the one calculated. Data at low magnetic fields were not compared with turbulent theory because a large number of electrons could diffuse to the anode before losing much energy, which would make the electron temperature difficult to estimate.

The residence time of an ion was largely set by the size of the equipment and corresponded to 100 or more plasma periods. The damping effect of neutrals should therefore be minimized. The excess of drift velocity above the critical value in the transition region may be explained in terms of amplification factor. To obtain the required amplification of noise, it may be necessary that the drift velocity exceed the critical value by a considerable amount. As indicated in reference 14, the turbulent diffusion expression of reference 4 also predicts results close to the experimental value. Thus, the effectiveness of more than one approach to a given problem is again demonstrated.

Noise was also monitored. Generally it covered a broad band of frequencies, although sometimes a clearly defined harmonic oscillation could be observed against the background noise. As would be expected for operation in the transition region, the noise increased with magnetic field strength.

CONCLUDING REMARKS

As the plasma changes from a laminar to a fully developed turbulent state, the electron mean free path should decrease from the laminar value to the minimum turbulent value, which is determined by energy-dissipating processes in the plasma. The transitional region is defined by mean

free paths between these two extremes. For the two-stream instability mechanism used herein, the onset of turbulence is predicted by the critical electron drift velocity. The expected minimum mean free path for this mechanism should be roughly the minimum plasma wavelength, which is set by a rapid increase in damping. In the transitional region, the electron drift velocity should tend toward the critical value.

The comparison of turbulent diffusion theory with experimental data is not conclusive, although it is also not contradictory. The uncertainties regarding the damping effects of neutrals, the time required to develop turbulence, and the excess above critical drift velocity required to maintain turbulence all preclude firm conclusions. Even with due regard for these uncertainties, though, the general experimental trends are apparently in agreement with those hypothesized.

There is a need for plasma data that are sufficiently complete and accurate to provide checks for turbulent plasma theories. Such data are also very important for engineering applications in a field as complex as this.

Lewis Research Center

National Aeronautics and Space Administration
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APPENDIX - SYMBOLS

B	magnetic field strength, webers/sq m (1 weber/sq m equals 10^4 gauss)
b	impact parameter for a 90° deflection, m
D_0	diffusion coefficient in absence of magnetic field, m^2/sec
$D_{ }$	diffusion coefficient parallel to a magnetic field, m^2/sec
D_{\perp}	diffusion coefficient transverse to magnetic field, m^2/sec
E	electric field strength, v/m
F	ion-plasma frequency, cps
F_0	ion-plasma frequency for short wavelengths, cps
f	electron-plasma frequency, cps
j	current density, amp/sq m
j_c	critical current density, amp/sq m
j_θ	drift current density in presence of magnetic field, amp/sq m
j_{\perp}	diffusion current density across magnetic field, amp/sq m
l	electron mean free path, m
l_D	Debye shielding distance, m
M	ion mass, kg
m	electron mass, 9.107×10^{-31} kg
n	electron or ion density (equal in a plasma), number/cu m
n_0	neutral density, number/cu m
Q	cross section, sq m
Q_d	distant or coulomb cross section, sq m
Q_n	near or large-deflection cross section, sq m
q	electronic charge, 1.602×10^{-19} coulomb

r	cyclotron radius, m
V	potential, v
\bar{V}	thermal potential or temperature (temperature in $^{\circ}\text{K}/11,600$), v
\bar{v}	root-mean-square (rms) electron velocity, m/sec
x	distance, m
α	ion temperature divided by electron temperature
β	critical electron drift ratio (critical drift velocity/rms velocity)
ϵ_0	dimensional constant, 8.855×10^{-12} farads/m
θ	angle between electron drift velocity and applied gradient (both transverse to a magnetic field)
Λ	Debye shielding distance divided by collision radius
λ	plasma wavelength, m
λ_{\min}	minimum plasma wavelength, m
σ_0	conductivity in absence of magnetic field, amp/v-m
σ_1	conductivity across magnetic field, amp/v-m

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- E-1619
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